

## 5. Diffraction by a Plane Grating

### Objective

To study the diffraction of light with a plane grating and to measure the wave length of various spectral lines in Hg lamp and the resolving power of the grating.

### Theory

A diffraction grating consists of large number of slits each of width  $b$  which are separated by distance  $d$  called the spatial period. A plane wave incident normally on a grating produces a diffraction pattern, the intensity variation  $I$  of which is given by

$$I = I_o \frac{\sin^2 \beta}{\beta^2} \frac{\sin^2 N\gamma}{\sin^2 \gamma} \quad (1)$$

Where  $2\beta = kbs \sin \theta_d$  is the phase difference between the Huygens secondary waves coming from extreme edges of a slit,  $2\gamma = kds \sin \theta_d$  is the phase difference between the Huygens secondary waves from corresponding points in any two adjacent slits,  $I_o$  is the intensity of the incident light,  $\theta_d$  is the diffraction angle,  $N$  is the no. of slits and  $k = 2\pi/\lambda$  is the magnitude of the wave vector ( $\lambda$  being the wavelength of light). Since the width of each slit in the grating is small, the effect of  $\beta$  can be neglected and the intensity variation can be written approximately as

$$I = I_o \frac{\sin^2 N\gamma}{\sin^2 \gamma} \quad (2)$$

From equation 2 it can be seen that maxima will occur whenever  $\gamma = m\pi$  where  $m$  is the diffraction order. This thus corresponds to

$$d \sin \theta_d = m\lambda \quad m=0, \pm 1, \pm 2 \dots \quad (3)$$

Equation 3 shows that light of different wavelengths get diffracted at different angle from the central maxima. It is this property of the grating that makes it an useful dispersion element in optics.

### Procedure

#### Measurement of various spectral lines in a mercury lamp.

Set the telescope for viewing the object at infinity and the collimator for the parallel beam of light as described in the lecture 2 of this module. Use the mercury lamp for this experiment. When all these initial adjustments are over, the grating may be mounted for normal incidence. To do this following steps are to be followed.

1. Align the collimator and the telescope in a straight line. Locate the image of the slit at the crosswire. Note down the angular readings. **Lock the prism table.**

2. Rotate the telescope by  $90^\circ$  and **lock it, Fig 1.**
3. Mount the grating on the grating table as shown in fig 2.



Fig 2 mounting of grating (this may be replaced by better photograph)

and rotate the table without changing the angle between the telescope and collimator till you see a reflected image of the slit at the telescope Fig 3. Note down the angular reading. This will make the angle of incidence on to the grating at  $45^\circ$ .

Fig 3. Grating mounted at  $45^\circ$ . C: Collimator, G: Grating, T: Telescope.

4. Now rotate the grating table by  $45^\circ$ , the sense of rotation should be in such a direction that the angle of incidence becomes zero. This will set the grating for normal incidence. Rotate the telescope till the new reading differs from the reading taken in step 3 by  $45^\circ$  to make it collinear with the collimator as shown in Fig 4. **Lock the table again** and note down this reading. This will be the initial **direct** reading.

Fig 4. Grating Aligned for normal incidence

**Did you understand the reason for the above steps? As the condition of maxima given in eq 3, is valid for the normal incidence and hence the grating is required to be set for normal incidence.**

5. With the slit width narrowed down sufficiently, find the angular locations of the telescope for different diffracted coloured images of the slit with respect to the direct reading. The differences between the angular readings for different colours and the direct reading give the angles of diffraction for various wavelengths. The grating period of  $d$  is given by the manufacturer usually in terms of number of lines per cm.

#### Analyses

1. Determine the emission wavelengths in the spectrum of the given line source, using Eq. (3).
2. Estimate the errors in the measurement of  $\lambda$ .

#### Measurement of resolving power of a grating

The resolving power of grating is given by  $\frac{\lambda}{\Delta\lambda}$ . Where  $\lambda$  is the wave length of a particular line and  $\Delta\lambda$  is the separation between the closet line which can be resolved by the given grating. From eq 3

$$\frac{\lambda}{\Delta\lambda} = \tan \theta_d \Delta\theta_d, \quad (4)$$

where  $\theta_d$  is the diffraction angle in the given order and  $\Delta\theta_d$  is the corresponding angular separation of the nearest line. Equation 4 can be used to measure the resolving power of a grating experimentally.

#### Procedure:

1. In the above experiment replace the mercury lamp with the **sodium lamp** and do the alignment fro the grating as described above.
2. Set the grating for first order for the sodium D lines. Measure the angular position for both the lines. The wave lengths for sodium d lines are 5890 nm and 5896nm.
3. measure the anglur positions in second order also.

#### Analyses

Calculate the angular separation  $\Delta\theta_d$  for both the orders and estimate  $\frac{\lambda}{\Delta\lambda}$  from eq 4.